1. (10 pts) If \( f(x) = \frac{\ln x}{2} + 1 \), what is \( f^{-1}(x) \)? What is its domain and range?

If \( y = \frac{\ln x}{2} + 1 \), then \( y - 1 = \frac{\ln x}{2} \), \( 2y - 2 = \ln x \), and \( x = e^{2y-2} \). Thus \( f^{-1}(x) = e^{2x-2} \). The domain is the set of all real numbers. The range is the domain of \( f(x) \), i.e., the set of positive numbers.

2. (10 pts) Compute \( \lim_{x \to \infty} \frac{(\ln x)^4}{x} \).

By L'Hôpital's rule,

\[
\lim_{x \to \infty} \frac{(\ln x)^4}{x} = \lim_{x \to \infty} \frac{4(\ln x)^3}{x} = \lim_{x \to \infty} \frac{12(\ln x)^2}{1} = \lim_{x \to \infty} \frac{24(\ln x)}{x} = \lim_{x \to \infty} \frac{1}{x} = 0.
\]

3. (16 pts) Evaluate \( \int_{0}^{1} \frac{2x^3}{(x^2 + 1)^2} \, dx \).

Write

\[
\frac{2x^3}{(x^2 + 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2},
\]

and solve for \( A, B, C, D \): \( A = 2 \), \( C = -2 \), \( B = D = 0 \).

Substituting \( u = x^2 + 1 \),

\[
\int_{0}^{1} \frac{2x^3}{(x^2 + 1)^2} \, dx = \int_{0}^{1} \frac{2x}{x^2 + 1} - \frac{2x}{(x^2 + 1)^2} \, dx = \int_{1}^{2} \frac{1}{u} - \frac{1}{u^2} \, du = \ln 2 - \frac{1}{2}.
\]
4. (16 pts) Evaluate $\int_0^{\pi/2} \cos^4 x \, dx$.

Use trig identities to write

\[
\int_0^{\pi/2} \cos^4 x \, dx = \frac{1}{4} \int_0^{\pi/2} (\cos 2x + 1)^2 \, dx \\
= \frac{1}{8} \int_0^{\pi/2} \cos 4x + 4 \cos 2x + 3 \, dx,
\]

and substitution to evaluate the integral and obtain $\frac{3\pi}{16}$.

5. (16 pts) Evaluate $\int \frac{\sqrt{x^2-4x}}{x-2} \, dx$.

Substitute $y = x-2$ to rewrite the integral as $\int \frac{\sqrt{y^2-2^2}}{y} \, dy$, and then substitute $y = 2 \sec u$ to obtain

\[
2 \int \tan^2 u \, du = 2 \int (\sec^2 u - 1) \, du = 2 \tan u - 2u + C \\
= \sqrt{y^2-2^2} - 2 \sec^{-1} y/2 + C = \sqrt{x^2-4x-2} \cos^{-1} \frac{2}{x-2} + C.
\]

6. (16 pts) Evaluate $\int e^{t^{1/3}} \, dt$.

Substitute $x = t^{1/3}$ to obtain $3 \int x^2 e^x \, dx$. Integrate by parts twice to obtain

\[
3 \int x^2 e^x \, dx = 3x^2 e^x - 6 \int xe^x = (3x^2 - 6x + 6)e^x + C \\
(3t^{2/3} - 6t^{1/3} + 6)e^{t^{1/3}} + C.
\]
7. (16 pts) Evaluate $\int x \sec^2 x \, dx$.

Integrate by parts to get

$$\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx = x \tan x - \ln |\sec x| + C.$$

BONUS (10 pts) Evaluate $\int \frac{dx}{3 + 5 \sin x}$.

Substitute $u = \tan(x/2)$ to obtain

$$\int \frac{dx}{3 + 5 \sin x} = \int \frac{2/3 \, du}{u^2 + (10/3)u + 1}.$$

Using partial fractions, we can rewrite this integral as

$$\int \frac{1/4 \, du}{u + 1/3} - \int rac{1/4 \, du}{u + 3} = \frac{\ln |\tan(x/2) + 1/3| - \ln |\tan(x/2) + 3|}{4} + C.$$