Math Circle Topics Spring 2009

1. Difference calculus
   (a) Differences of polynomial sequences
   (b) Summation of polynomial sequences
   (c) Analogy with the continuous case

2. Triangles
   (a) Congruence conditions
   (b) Sum of internal and external angles
   (c) Proportions and the “rule of three”
   (d) Similar triangles
   (e) Right triangles
   (f) Pythagorean theorem
   (g) The equation of a circle

3. Trigonometry
   (a) Similar triangles and estimating heights
   (b) Trig functions and right triangles
   (c) The $30 - 60 - 90$ and $45 - 45 - 90$ triangles
   (d) Trig functions and the unit circle
   (e) $\cos^2 x + \sin^2 x = 1$

4. Complex numbers
   (a) Addition, subtraction, multiplication
   (b) Coordinate geometry and the complex plane
   (c) The square root of $i$
   (d) Multiplication by $i$ as rotation
5. Quadratic Equations
   (a) Determining \( x \) and \( y \) such that \( x + y = 10 \) and \( xy = 21 \)
   (b) Solving \( x^2 - 10x + 21 = 0 \) versus factoring \( x^2 - 10x + 21 \)
   (c) The “quadratic formula” for \( x^2 - ax + b \)
   (d) Solving by completing the square

6. Symbolic Logic
   (a) Truth tables
   (b) Binary arithmetic
   (c) Logic and digital circuitry

7. Modular Arithmetic
   (a) Addition, subtraction, multiplication
   (b) Casting out nines
   (c) Day of the week computations
   (d) Chinese Remainder Theorem
   (e) Euclidean algorithm
   (f) Arithmetic progressions (mod \( m \))
   (g) Solving \( an \equiv 1 \pmod{m} \)
   (h) Explicit solutions of simultaneous congruences
   (i) Geometric progression (mod \( m \))
   (j) Fermat’s Little Theorem
   (k) Why the decimal for \( 1/7 \) has period 6

8. Simultaneous Linear Equations
   (a) Intersecting lines and solving two equations in two unknowns
   (b) Word problems
   (c) Solving by choosing linear combinations of the equations
   (d) Matrices and row reduction
9. **Regular Polygons and Regular Polyhedra**
   (a) Plane tesselations by regular polygons
   (b) The five platonic solids
   (c) Euler’s formula

10. **Probability**
    (a) Poker hands
    (b) Independence
    (c) Dice probabilities
    (d) Coin tossing and Pascal’s triangle
    (e) Weighted coins and Mendelian genetics

11. **Approximating Square Roots**
    (a) Isoperimetric inequality for rectangles
    (b) Arithmetic-geometric mean inequality
    (c) Iterating $x \mapsto \frac{x + a/x}{2}$
    (d) Newton’s method and $x \mapsto x + \frac{a-x^2}{2x}$.

12. **Volume Formulas**
    (a) Defining volume and area with grids
    (b) Cavalieri’s principle
    (c) Prisms and cylinders
    (d) Areas of cross-sections of pyramids
    (e) Partitioning a cube into six pyramids
    (f) Volumes of pyramids and cones
    (g) Ball = Cylinder - Cone
    (h) Area under parabolas and integral calculus