Math S212 Exam 3 Name:

For each convergence problem, you should not only indicate whether the sum in question converges or diverges. You should also give a reason, such as "p-test" or "ratio test" and substantiating details if necessary. (For instance, if you say "limit comparison test", you should at least indicate the series you are comparing to and how you know whether that series converges or diverges.)

1. (10 pts) Evaluate the sum $3/2 + 1 + 2/3 + (2/3)^2 + (2/3)^3 + \cdots$ or indicate if it does not converge.

2. (10 pts) Evaluate the sum

   $$(\sin 0 - \sin \pi/2) + (\sin \pi/2 - \sin \pi) + (\sin \pi - \sin 3\pi/2) + (\sin 3\pi/2 - \sin 2\pi) + \cdots$$

   or indicate if it does not converge.

3. (10 pts) Does the series $\frac{10}{1!} - \frac{10^2}{2!} + \frac{10^3}{3!} - \cdots$ converge, and if so, is the convergence conditional or absolute?

4. (10 pts) Does $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$ converge or diverge?

5. (10 pts) Does $\sum_{n=2}^{\infty} \frac{\sqrt{n+1}}{n(n-1)}$ converge or diverge?

6. (10 pts) Does $\frac{1}{2\ln 2} - \frac{1}{3\ln 3} + \frac{1}{4\ln 4} - \cdots$ converge, and if so, is the convergence conditional or absolute?

7. (10 pts) Does $\sum_{n=1}^{\infty} \left(\frac{\ln n}{n}\right)^2$ converge or diverge?

8. (10 pts) Does $\sum_{n=1}^{\infty} \frac{2^n - 1}{n^n}$ converge or diverge?

9. (10 pts) Does $\sum_{n=1}^{\infty} \frac{2+\sin n}{n}$ converge or diverge?

10. (10 pts) The sequence $a_n = \frac{n}{n+1}$ converges to 1. That means, for all $\epsilon > 0$, there exists N such that for all $n \geq N$, $|a_n - 1| < \epsilon$. Find a formula for N in terms of ϵ which shows this is true.

BONUS (10 pts) Does $\sum_{n=1}^{\infty}(1 - \cos(1/n))$ converge or diverge?