

Trig Identities

$$\sin^2 x + \cos^2 x = 1. \quad \tan^2 x + 1 = \sec^2 x. \quad \cot^2 x + 1 = \csc^2 x.$$

$$\sin(-x) = -\sin x. \quad \cos(-x) = \cos x. \quad \tan(-x) = -\tan x.$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y. \quad \cos(x + y) = \cos x \cos y - \sin x \sin y.$$

$$\sin 2x = 2 \sin x \cos x. \quad \cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x.$$

$$\frac{d \sin x}{dx} = \cos x. \quad \frac{d \cos x}{dx} = -\sin x. \quad \frac{d \tan x}{dx} = \sec^2 x.$$

$$\frac{d \csc x}{dx} = -\cot x \csc x. \quad \frac{d \sec x}{dx} = \tan x \sec x. \quad \frac{d \cot x}{dx} = -\csc^2 x.$$

Problems

1. (10 pts) If $f(x) = \frac{\ln x}{2} + 1$, what is $f^{-1}(x)$? What is its domain and range?

2. (10 pts) Compute $\lim_{x \rightarrow \infty} \frac{(\ln x)^4}{x}$.

3. (16 pts) Evaluate $\int_0^1 \frac{2x^3}{(x^2+1)^2} dx$.

4. (16 pts) Evaluate $\int_0^{\pi/2} \cos^4 x dx$.

5. (16 pts) Evaluate $\int \frac{\sqrt{x^2-4x}}{x-2} dx$.

6. (16 pts) Evaluate $\int e^{t^{1/3}} dt$.

7. (16 pts) Evaluate $\int x \sec^2 x \, dx$.

BONUS (10 pts) Evaluate $\int \frac{dx}{3+5 \sin x}$.