Math Circle Topics Spring 2009

1. **Difference calculus**
   (a) Differences of polynomial sequences
   (b) Summation of polynomial sequences
   (c) Analogy with the continuous case
2. **Triangles**
   (a) Congruence conditions
   (b) Sum of internal and external angles
   (c) Proportions and the "rule of three"
   (d) Similar triangles
   (e) Right triangles
   (f) Pythagorean theorem
   (g) The equation of a circle
3. **Trigonometry**
   (a) Similar triangles and estimating heights
   (b) Trig functions and right triangles
   (c) The $30-60-90$ and $45-45-90$ triangles
   (d) Trig functions and the unit circle
   (e) $\cos^2 x + \sin^2 x = 1$
4. **Complex numbers**
   (a) Addition, subtraction, multiplication
   (b) Coordinate geometry and the complex plane
   (c) The square root of i
   (d) Multiplication by i as rotation

5. **Quadratic Equations**

   (a) Determining x and y such that $x + y = 10$ and $xy = 21$

   (b) Solving $x^2 - 10x + 21 = 0$ versus factoring $x^2 - 10x + 21$

   (c) The "quadratic formula" for $x^2 - ax + b$

   (d) Solving by completing the square

6. **Symbolic Logic**

   (a) Truth tables

   (b) Binary arithmetic

   (c) Logic and digital circuitry

7. **Modular Arithmetic**

   (a) Addition, subtraction, multiplication

   (b) Casting out nines

   (c) Day of the week computations

   (d) Chinese Remainder Theorem

   (e) Euclidean algorithm

   (f) Arithmetic progressions (mod m)

   (g) Solving $an \equiv 1 \pmod{m}$

   (h) Explicit solutions of simultaneous congruences

   (i) Geometric progression (mod m)

   (j) Fermat's Little Theorem

   (k) Why the decimal for 1/7 has period 6

8. **Simultaneous Linear Equations**

   (a) Intersecting lines and solving two equations in two unknowns

   (b) Word problems

   (c) Solving by choosing linear combinations of the equations

   (d) Matrices and row reduction

9. **Regular Polygons and Regular Polyhedra**

    (a) Plane tesselations by regular polygons

    (b) The five platonic solids

    (c) Euler's formula

10. **Probability**

    (a) Poker hands

    (b) Independence

    (c) Dice probabilities

    (d) Coin tossing and Pascal's triangle

    (e) Weighted coins and Mendelian genetics

11. **Approximating Square Roots**

    (a) Isoperimetric inequality for rectangles

    (b) Arithmetic-geometric mean inequality

    (c) Iterating $x \mapsto \frac{x+a/x}{2}$

    (d) Newton's method and $x \mapsto x + \frac{a-x^2}{2x}$.

12. **Volume Formulas**

    (a) Defining volume and area with grids

    (b) Cavalieri's principle

    (c) Prisms and cylinders

    (d) Areas of cross-sections of pyramids

    (e) Partitioning a cube into six pyramids

    (f) Volumes of pyramids and cones

    (g) Ball = Cylinder - Cone

    (h) Area under parabolas and integral calculus