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1. (10 pts) What continuous growth rate would make the world population grow from 6 billion in the year 2000 to 12 billion in the year 2100? (You may leave an unevaluated natural logarithm in the answer.) What is the formula for population as a function of t, the number of years since 2000?

Let $f(t)$ be population in billions in the year $2000+t$. Then $f(t) = P_0 e^{kt}$. As $f(0) = 6$, $P_0 = 6$. As $f(100) = 12$, $e^{100k} = \frac{12}{6} = 2$, so $100k = \ln 2$, and $k = \frac{\ln 2}{100}$ is the continuous growth rate. The population function is therefore

$$6e^{\frac{\ln 2}{100}t} \textit{ billion.}$$

2. (10 pts) Consider a function $f(x)$ given by the following table:

x	5	10	15	20	25	30
f(x)	15	95	225	405	635	915

(a) Estimate $f'(10)$ and $f'(15)$.
(b) Estimate $f''(10)$.

For (a), $f'(10) \approx \frac{f(15)-f(10)}{15-10} = 26$, and $f'(15) \approx \frac{f(20)-f(15)}{20-15} = 36$. For (b), $f''(10) \approx \frac{f'(15)-f'(10)}{15-10} = 2$.

3. (10 pts) A car makes a one hour journey in such a way that in t hours it travels $30t^2$ miles. What is its speed (in miles per hour) after 30 minutes? What is its acceleration (in miles per hour per hour) at the beginning of the trip? (Be careful of units.)

Converting into miles, 30 minutes becomes $t = 1/2$. Speed (or velocity) is the derivative of $f(t) = 30t^2$ which is $f'(t) = 60t$, which at $t = 1/2$ is 30 M.P.H.. Acceleration is the derivative of velocity: $f''(t) = 60$, so acceleration is 60 miles per hour per hour throughout the journey.

4. (20 pts) Compute $\frac{dy}{dx}$:

(a) $y = (x^2+1)^5$

(b) $y = \ln \frac{x}{x-1}$

(c) $\frac{y=1}{\sqrt{2x+1}}$

(d) $y = 2^x \sin x$

(a) Chain rule and power rule:

$$\frac{dy}{dx} = 5(x^2+1)^4(2x) = 10x(x^2+1)^4.$$

(b) The simplest thing is to write $y = \ln x - \ln(x-1)$, *so*

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x-1} = \frac{-1}{x(x-1)}.$$

(c) The simplest thing is to write $y = (2x+1)^{-1/2}$, *so*

$$\frac{dy}{dx} = (-1/2)(2x+1)^{-3/2}(2) = -(2x+1)^{-3/2} = \frac{-1}{(2x+1)^{3/2}} = \frac{-1}{(2x+1)\sqrt{2x+1}}.$$

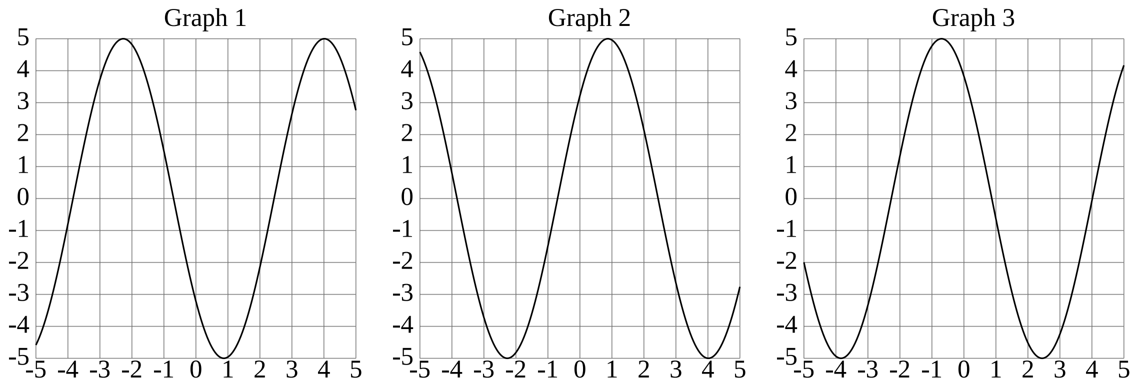
(d) Product rule and exponential rule:

$$\frac{dy}{dx} = (\ln 2\ 2^x)\sin x + 2^x \cos x = x^2(\ln 2\ \sin x + \cos x).$$

5. (10 pts) Suppose that $f(x)$ is a function with $f(10) = 5$ and $f'(10) = -1$. Estimate $f(9)$.

If $y = f(x)$, $\frac{\Delta y}{\Delta x} \approx f'(x)$. *Applying this when* $x = 10$, $y = 5$, *and* $\Delta x = -1$, *we get* $\Delta y \approx 1$, *so* $f(9) = f(x + \Delta x) = y + \Delta y \approx 6$.

6. (10 pts) The following diagram shows the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$ for some function $f(x)$. Which is which?



Graph 1 draws a function decreasing at zero and Graphs 2 and 3 are positive at zero, so neither one shows the derivative of the function in Graph 1. Therefore Graph 1 can be neither $y = f(x)$ nor $y = f'(x)$ and must be $y = f''(x)$. The function in Graph 3 is also decreasing at 0, so Graph 2 cannot show its derivative. It follows that Graph 2 shows $y = f(x)$ and Graph 3 shows $y = f'(x)$.

7. (10 pts) The quantity of a drug in a patient's body t hours after it is administered is $100e^{-t}$ milligrams. What is the rate at which the drug level is changing after one hour? Remember to include units in your answer.

Rate of change is given by the derivative, which in this case is $100(-1)e^{-t} = -100e^{-t}$ evaluated at $t = 1$. This gives $\frac{-100}{e}\frac{mg.}{hr.}$ or $\frac{-100}{e}$ milligrams per hour.

8. (10 pts) Find the equation of the tangent line to $y = \ln(2x - 1)$ at $(1, 0)$.

To compute slope, evaluate $\frac{dy}{dx} = \frac{1}{2x-1}2 = \frac{2}{2x-1}$ at $x = 1$. This gives a slope of 2 and a tangent line of the form $y = 2x + b$. Since $y = 0$ when $x = 1$, $b = -2$, and the line is $y = 2x - 2$.

9. (10 pts) The cost function for a certain manufacturer is $C(q) = 20000 + 1000\ln(q + 1)$. What is the marginal cost at a production level of $q = 49$?

The marginal cost is the derivative of the cost function $C'(q) = \frac{1000}{q+1}$. Therefore, the marginal cost at $q = 49$ is $\frac{1000}{50} = 20$.