

## SET 7

- (1) Evaluate the following infinite product:

$$\prod_{n=0}^{\infty} (1 + 2^{-2^n}).$$

- (2) Point  $P$  is inside triangle  $ABC$ , and the areas of the triangles  $ABP$ ,  $BCP$ , and  $CAP$  are all equal. Prove that  $P$  lies on every line connecting a vertex of the triangle to the midpoint of the opposite side.
- (3) The square  $[0, n] \times [0, n]$  has  $(n + 1)^2$  lattice points. How many ways can each of these points be colored red or blue, so that each unit square with vertices among the lattice points in the square has exactly two red vertices?