

### SET 4

- (1) Prove that every integer greater than 33 is the sum of distinct triangular numbers (i.e., distinct terms of the sequence  $1, 3, 6, 10, 15, \dots$ ).
- (2) Let  $F_0 = 1$ ,  $F_1 = 1$ , and  $F_{n+2} = F_{n+1} + F_n$  for  $n \geq 0$ . Evaluate

$$\sum_{i=0}^n \frac{F_i}{2^i}.$$

- (3) For each positive integer  $n$ , let  $a_n = -\log n + \sum_{i=1}^n 1/i$ . Prove that the sequence  $a_1, a_2, \dots$  is convergent.