

## SET 10

- (1) Show that if  $a$  and  $b$  are positive real numbers,

$$\lim_{n \rightarrow \infty} \left( \frac{a^{1/n} + b^{1/n}}{2} \right)^n = \sqrt{ab}.$$

- (2) Let  $a_{n+1} = 2 \log_2 a_n$ . Show that if  $a_0 \geq 2$ , the sequence  $a_0, a_1, a_2, \dots$  converges, and express the limit in terms of  $a_0$ .
- (3) Show that if four distinct points of the curve  $y = 2x^4 + 7x^3 + 3x - 5$  are collinear, then their average  $x$ -coordinate is some constant  $k$ . Find  $k$ .