SET 1

(1) Let $a_1, a_2, \ldots, a_n$ be distinct positive integers such that

$$a_1 + a_2 + \cdots + a_n = 30.$$

Find the largest possible value of $a_1 a_2 \cdots a_n$. Justify your answer.

(2) Let $\{x\}$ be the fractional part of a real number x, i.e., $\{x\} = x - \lfloor x \rfloor$, where $\lfloor x \rfloor$ is the integer part of x. Suppose α is a positive real number such that $\{\alpha n\} < \frac{1}{2}$ for all positive integers n. Show that α must be an integer.

(3) Prove that if $s > 1$ then the triple integral

$$\int_1^\infty \int_1^\infty \int_1^\infty \frac{dx\, dy\, dz}{(x^3 + y^3 + z^3)^s}$$

converges.